Calculation Policy Mathematics

Working Collaboratively

Aston Fields Middle School

'Striving for Excellence: Learning for Life'

Finstall First School

'Learning Together, Preparing for Life'

Hanbury C. of E. First School

'Flourishing Through Love and Nurture'

Stoke Prior First School

'A small school with big ideas, making learning memorable!'

About our Calculation Policy

The following Calculation Policy has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics. Finstall First, Hanbury C. of. E. First, Stoke Prior First and Aston Fields Middle Schools have worked collaboratively to ensure the progression of learning calculations across the first schools and into the middle school for the end of Key Stage 2.

Please note that early learning in number and calculation, taught in Reception, follows the Early Years Foundation Stage.

Age Related Expectations

The Calculation Policy is organised according to typical age-related expectations, as set out in the National Curriculum 2014. **However, it is vital that pupils are taught according to the level of ability and understanding they are currently working at.** They should only progress onto the next stage when they are ready and they feel secure enough to move on.

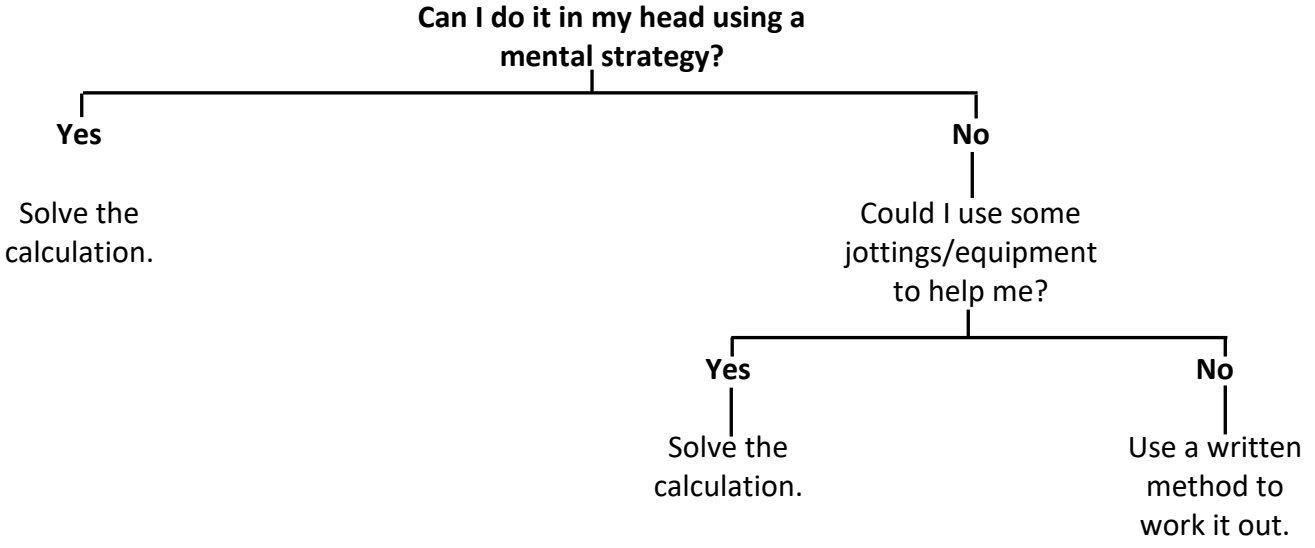
Children will be given time to develop fluency and automaticity in using the differing methods seen within this document and may start their learning using manipulatives to support their development and deepen their understanding.

Providing a Context for Calculation

It is important that any type of calculation is given a real-life context. A relevant problem-solving approach helps to build children's understanding of the purpose of calculation and helps them to recognise when to use certain operations and methods when faced with problems.

Choosing a Calculation Method

Children need to be taught and encouraged to use the following process when deciding which approach they will take in order to solve a calculation. This will help to ensure that they select the most appropriate method for the numbers involved.



Addition

EYFS: Children to have a deep understanding of number to 10.

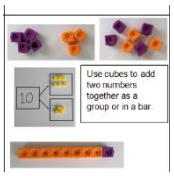
Key Skills for addition in EYFS:

- Subitise up to 5.
- Automatically recall number bonds to 5 and some number bonds to 10.
- Explore and represent patterns within numbers up to 10 including double facts.
- Children to verbally count beyond 20, recognising the pattern of the counting system.
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity.

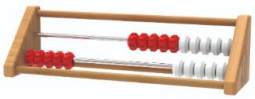
Key Vocabulary

put together, add, altogether, total, more than, equal to, equals, double, most, count on, subitise, same, different.

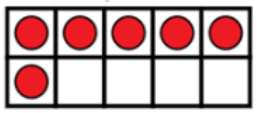
- A wide range of concrete counting equipment.
    - Everyday objects such as: small vehicles, small animals, jewels and teddy bears.
    - Bead strings
    - Cubes



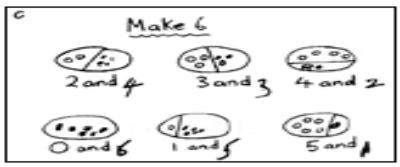
Rekenrek



Ten frames

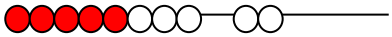


They should also be encouraged to use a range of representations to help them visualise Maths in a pictorial way. This will allow them to see numbers in different contexts.



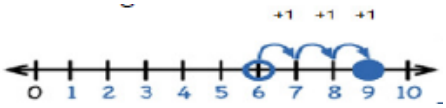
- Rekenreks
- Place Value Counters
- Cubes
- Bead strings can be used to illustrate addition

8+2=10



They use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line.

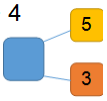
6+3=9



- Numicon to support number sentence formation
- Number lines
- Part-Whole Models

4 + 3 = 7

10= 6 + 4



Number will be embedded throughout the curriculum and opportunities for explorative play within provision.

Children at the expected level of development will:

Number

- Have a deep understanding of number to 10, including the composition of each number;
- Subitise (recognise quantities without counting) up to 5;
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Numerical Patterns

- Verbally count beyond 20, recognising the pattern of the counting system;
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Addition

Year 1: Add with numbers up to 20.

Key Vocabulary

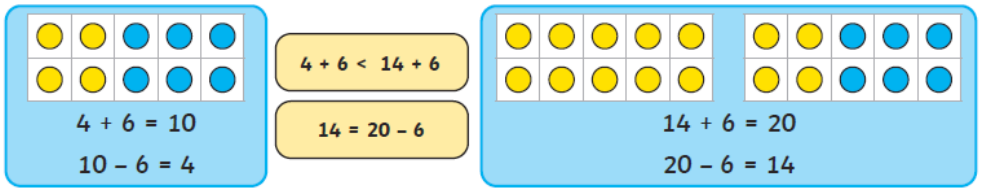
Add, more, plus, and, make, altogether, total, equal to, equals, double, count on, number line

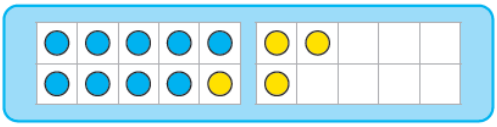
They should start their learning journey using manipulatives such as:

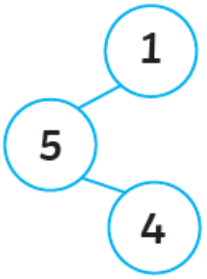
- A wide range of counting equipment.
  - Everyday objects such as: small vehicles, small animals, jewels and teddy bears.
  - Bead strings
  - Cubes

They should also be encouraged to use a range of representations to help them visualise Maths in a pictorial way. This will allow them to see numbers in different contexts.

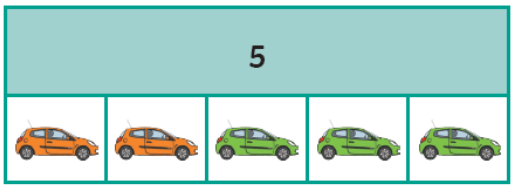
- Dienes
- Place Value Counters
- Cubes
- Numicon
- Hundred Squares
- Number lines
- Part-Whole Models
- Bar Models
- Rekenrek

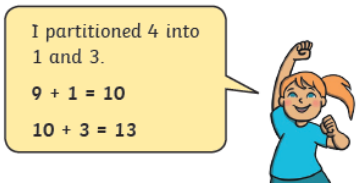






5 = 1 + 4





- Use numbered lines to add by counting on in ones. Encourage children to start with the **larger** number and count on.

They should be able to:

- Read and write the addition (+) and equals (=) signs within number sentences.

- Interpret addition number sentences and solve missing number problems using concrete objects and number line addition to solve them:

8 + 3 = ☐ 15 + 4 = ☐ 5 + 3 + 1 = ☐ ☐ + ☐ = 6

This builds on from prior learning of adding by combining two sets of objects into one group in Foundation Stage.

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

Addition

Year 2: Add with 2-digit numbers. Developing mental fluency with addition and place value involving 2-digit numbers, then establishing more formal methods.

Key Skills for addition in Year 2:

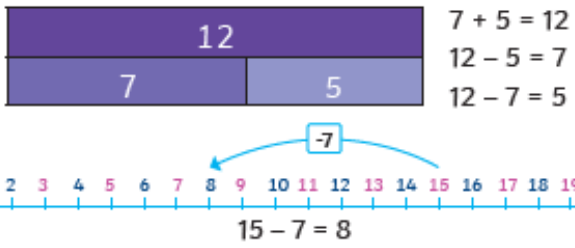
- Recognise the place value of each digit in a two-digit number.
- Add a 2-digit number and ones (E.g. 27 + 6)
- Add a 2-digit number and tens (E.g. 23 + 40)
- Add pairs of 2-digit numbers (E.g. 35 + 37)
- Add three, single-digit numbers (E.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law)
- Recall bonds to 20 and bonds of 10 to 100 (E.g. 70 + 30)
- Solve addition problems in a range of contexts (numbers, quantities and measures) using concrete objects, pictorial representations and by applying mental and written methods.
- To use the symbols + and =.

Key Vocabulary

Add, total, make, plus, sum, more, altogether, column addition, estimate, inverse operation, solve problems, number facts, place value, tens, ones, exchange, regroup

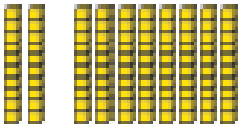
Children may start their learning through the use of concrete and pictorial methods. They may use a range of equipment, such as:

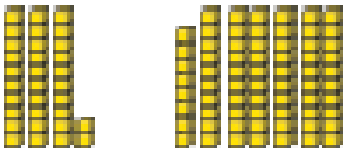
- Dienes
- Place Value Counters
- Bar Models
- Part-Whole Models
- Cubes
- Numicon
- Hundred Squares
- Rekenrek



2 + 8 = 10

so 20 + 80 = 100





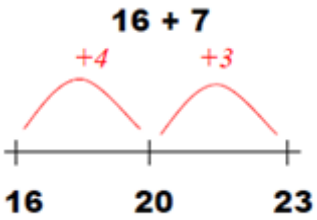
32 + 68 = 100

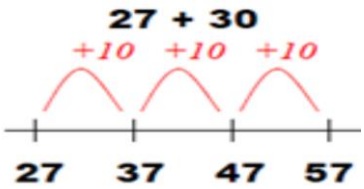
3 tens and 2 ones + 6 tens and 8 ones

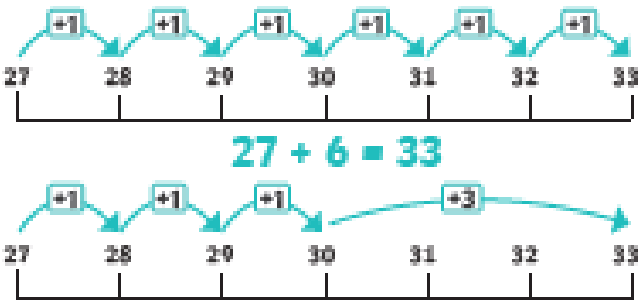
= 9 tens and 10 ones = 10 tens =one hundred

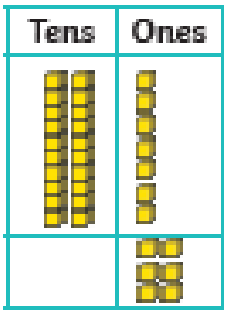
9 + 5 + 3 = 17

Add 2-digit numbers and ones and add 2-digit numbers and tens.

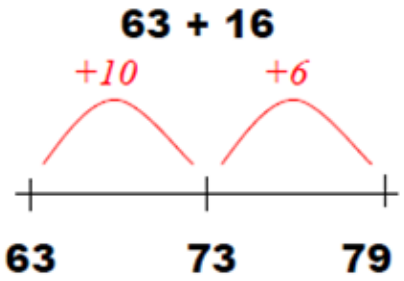


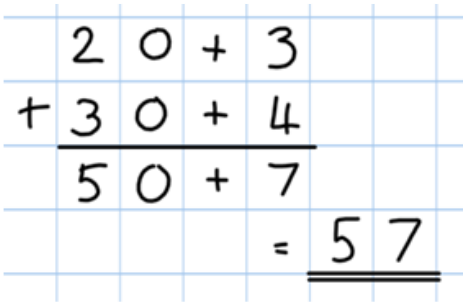






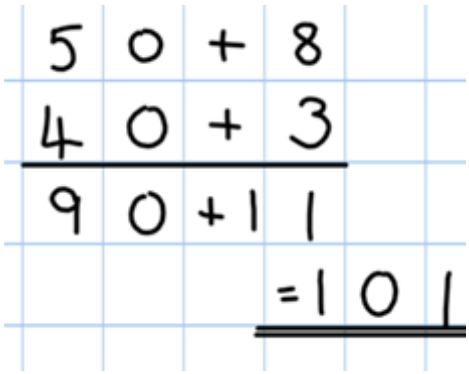
Add pairs of 2-digit numbers, moving to the partitioned column method <u>when secure</u> (adding tens and units):



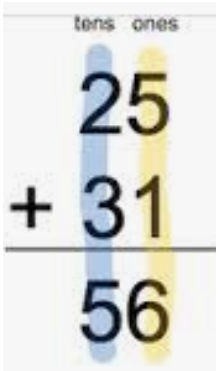


STEP 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).



STEP 3: Children who are confident and accurate with place value move to:



Addition

Year 3: Add numbers with up to 3-digits.

Key Skills for addition in Year 3:

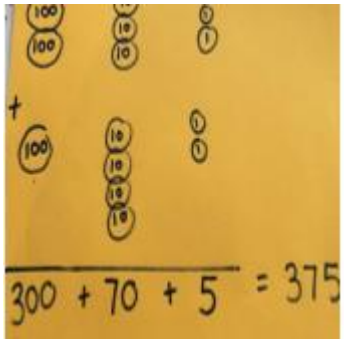
- Read and write numbers to 1000 in numerals and words.
- Recognise the place value of each digit in a three-digit number.
- Add 2-digit numbers mentally, including those exceeding 100.
- Add numbers with up to 3-digits using the formal method of columnar addition.
- Add a three-digit number and ones mentally. (E.g. 175 + 8)
- Add a three-digit number and tens mentally. (E.g. 249 + 50)
- Add a three-digit number and hundreds mentally. (E.g. 381 + 400)
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Continue to practise a wide range of mental addition strategies. E.g. Number bonds, adding the nearest multiple of 10 and 100 and adjusting, using near doubles, partitioning and recombining.

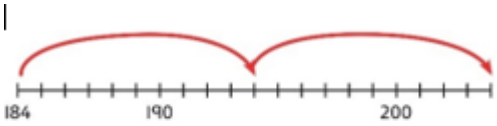
Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, regroup/exchange, expanded, compact

Children will continue to learn through the use of concrete and pictorial methods. They may use a range of equipment or approaches such as:

- Dienes
- Place Value counters
- Bar models
- Part-whole models
- Numicon
- Hundred Squares
- Number lines
- Cubes



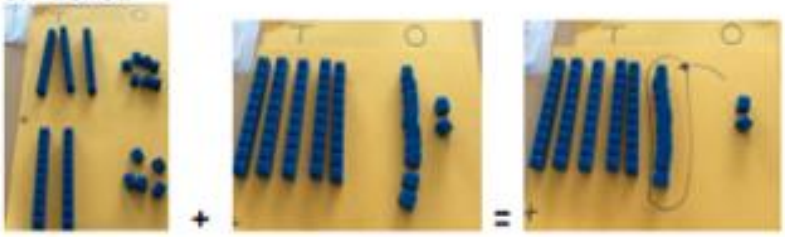


184 + 20 = ?

I can count in 10s … 194 … 204
184 + 20 = 204

352 + 134 = 486

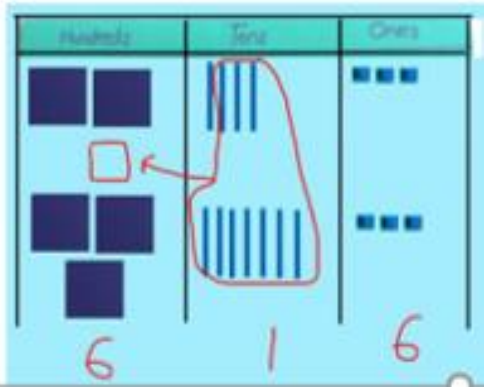
Modelled using Base 10 and place value counters-
Add the ones together first then the tens.
37+25=62



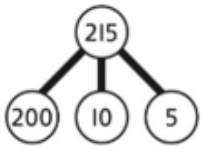
Modelled using Base 10
Children to understand that the highest amount in each column is 9 so sometimes exchange into the next column is necessary.

Children know to exchange ten 1s for a ten and ten 10s for a hundred.

243+ 373 = 616



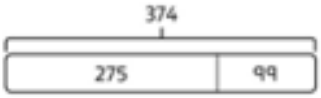
Represent the parts of numbers to 1,000 using a part-whole model.



215 = 200 + 10 + 5

Children understand and create bar models to represent addition problems.

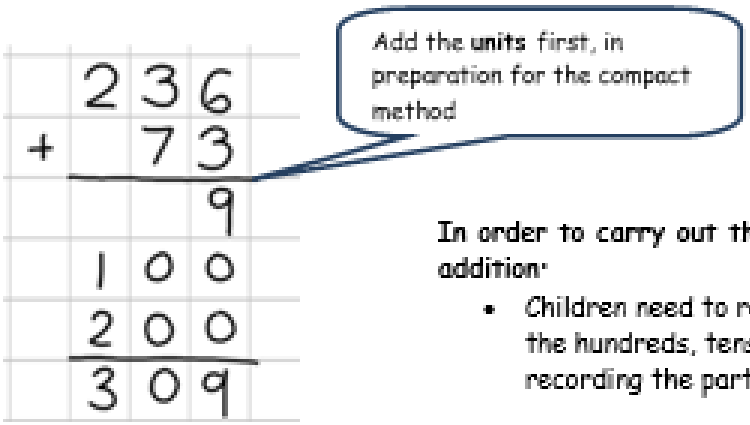
275 + 99 = ?



275 + 99 = 374

Children, who didn't move to Step 3 in Year 2, should continue to use the expanded method until their understanding of place value is secure, before moving to the compact method.

Step 1



In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units without recording the partitioning.
- Pupils need to be able to add in columns.

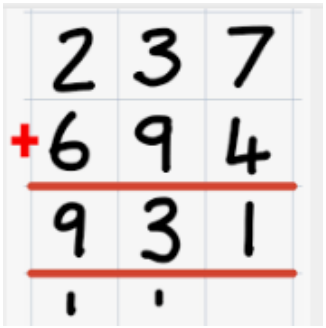
Move to the compact column addition method, with 'carrying':

Step 2

Children should be introduced to the compact column method as a continuation from Year 2, where no 'carrying' is required in the addition.

When modelling, pupils should be reminded that the actual value is '3 ones, + 5 ones = 8 ones, 2 tens + 1 ten = 3 tens (not 2 + 1 = 3) and 6 hundred + 2 hundreds = 8 hundreds (not 6 + 2 = 8)

Step 3



Children should then be introduced to 'carrying' for the first time when using the compact method with numbers up to 3-digits.

Addition

Year 4: Add with 4-digit numbers.

Key Skills for addition in Year 4:

- Read and write numbers
- Recognise the place value of each digit in a four-digit number.
- Select the most appropriate method: mental, jottings or written and explain why.
- Add numbers with up to 4-digits using the formal method of columnar addition.
- Estimate and use inverse operations to check answers.
- Solve addition two-step problems in contexts, deciding which operations to use and why.
- Continue to practise a wide range of mental addition strategies. E.g. Number bonds, adding the nearest multiple of 10, 100 and 1000 and adjusting, using near doubles, partitioning and recombining.

Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, plus, addition, column, tens boundary, hundreds boundary, thousands boundary, ten thousand, increase, vertical, regroup/exchange, expanded, compact, thousands, hundreds, digits, inverse

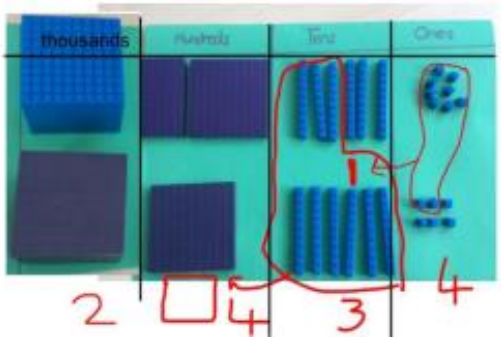
Children will continue to learn through the use of concrete and pictorial methods. They may use a range of equipment or approaches such as:

- Dienes
- Place Value Counters
- Bar models
- Part-whole models
- Cubes
- Numicon
- Hundred Squares
- Number Lines

<u>Modelled using Base 10 (Dienes) Equipment</u>

Children to understand that the highest amount in each column is 9 so sometimes exchange into the next column is necessary. Children understand that they can exchange ten 1s for a ten and ten 10s for a hundred and ten 100s for a thousand. Children begin to understand multi exchange where exchange is needed in more than one column.

1268+ 1166 = 2434

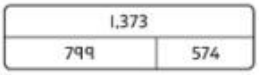


Use pictorial representations to add numbers up to 4 digits. Children will use images to represent the place value. If exchanging is needed, this will be shown below the line. This leads to greater understanding when using the formal written method as the children know what the digit below the line represents.

Place value counters can be used.

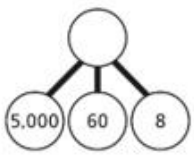
3,242 + 2,213 = 5,455

Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.



Th	H	T	O
	7	9	9
+	5	7	4
1	3	7	3

Understand partitioning of 4-digit numbers, including numbers with digits of 0.

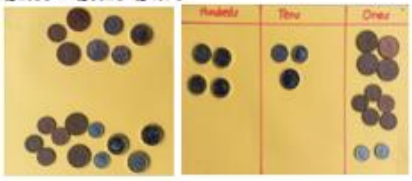


5,000 + 60 + 8 = 5,068

Use physical objects to solve simple measure and money problems.

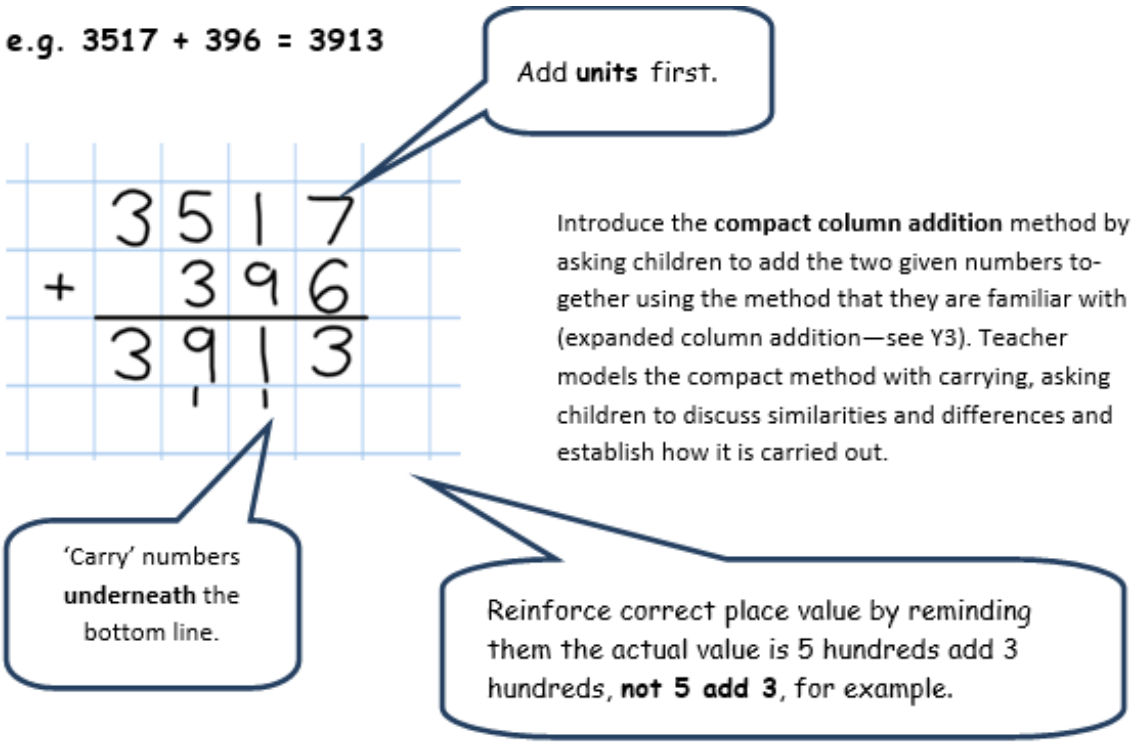
Children will gather then organise the amount required. Using the place value chart, children will then solve the calculation.

£1.55 + £3.18=£4.73



Compact Method

e.g. 3517 + 396 = 3913

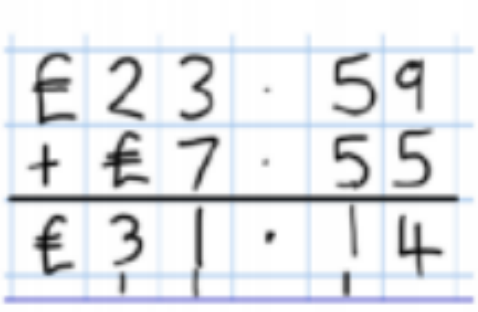


Use and apply this method to money and measurement values.

Use and apply this method to money and measurement values.

Record as a written calculation

<u>Condensed columnar addition</u>
Children should line the decimals correctly under one another, considering place value.



Addition

Year 5: Add numbers with more than 4 digits including money, measures and decimals (with different numbers of decimal places)

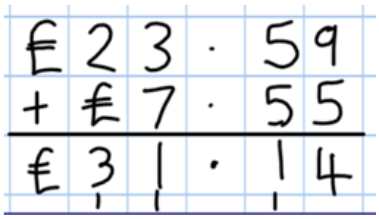
Key Skills for addition in Year 5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies. E.g. Add the nearest multiple of 10, 100 and 1000 and adjust; use near doubles, inverse, partitioning and recombining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Add numbers with more than 4-digits using formal written methods of columnar addition.

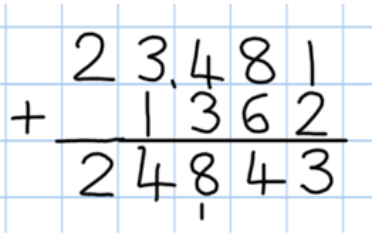
Key Vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, plus, addition, column, tens boundary, hundreds boundary, thousands boundary, ten thousands barrier, millions increase, vertical, regroup/exchange, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

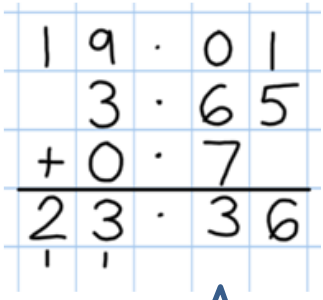
In Year 5 pupils will carry on building on the methods they have been learning in previous years.



The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer



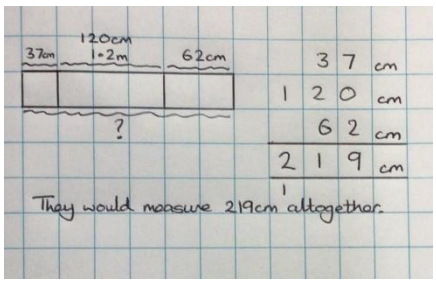
Numbers should exceed 4 digits.



Pupils should be able to add more than two values, carefully aligning place value columns.

Say '6 tenths add 7 tenths' to reinforce place

Empty decimal places can be filled with zero to show the place value in each column.



Pupils can use pictorial methods to help them solve addition word problems.

Sarah has 3 pieces of string the first is 37cm, the second is 1.2m and the third is 62cm. How much would they measure altogether?

Children should:

Understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

Addition

Year 6: Add several numbers of increasing complexity.

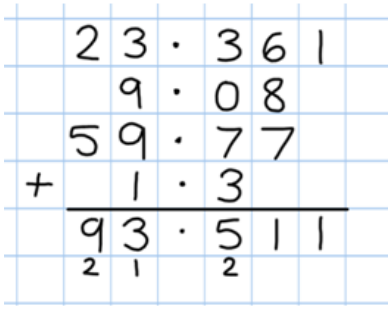
Key Skills for addition in Year 6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

Key Vocabulary

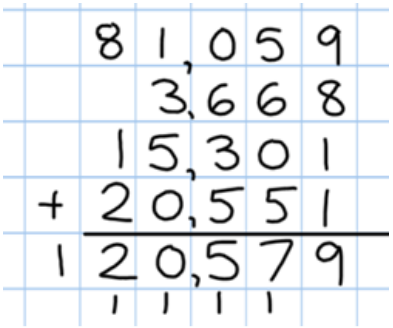
Add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, ones, partition, plus, addition, column, tens boundary, hundreds boundary, thousands boundary, ten thousands boundary, million increase, vertical, regroup/exchange, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

In Year 6 pupils will carry on building on the methods they have been learning in previous years.

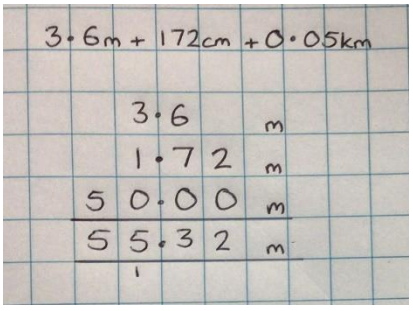


- Adding several numbers with different numbers of decimal places (including money and measures):
- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.

Empty decimal places can be filled with zero to show the place value in each column (place holder).



- Adding several numbers with more than 4 digits



- Adding mixed units of measures.

Subtraction

EYFS: Children to have a deep understanding of number to 10, the relationships between them and the patterns within those numbers.

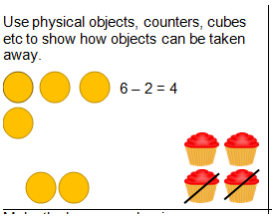
Key Skills for subtraction in EYFS:

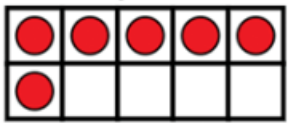
- Automatically recall number bonds to 5 (including subtraction facts).
- Explore and represent patterns within numbers up to 10.
- Children to verbally count beyond 20, recognising the pattern of the counting system.
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity.

Key Vocabulary

Take away, subtract, fewer than, less than, least.

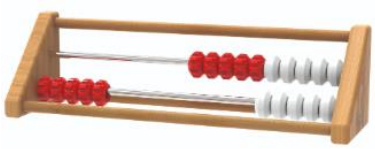
- A wide range of concrete counting equipment.
    - Everyday objects such as: small vehicles, small animals, jewels and teddy bears.
    - Bead strings
    - Cubes
    - Ten frames

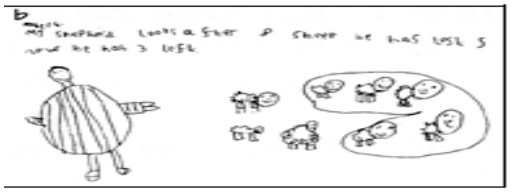




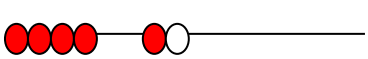
They should also be encouraged to use a range of representations to help them visualise Maths in a pictorial way. This will allow them to see numbers in different contexts.

- Rekenreks
- Place Value Counters
- Cubes



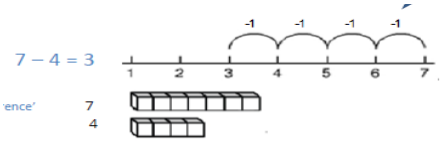


Bead strings - including bridging through ten by counting back 3 then counting back 2.



- Numicon to support number sentence formation
- Part-Whole Models
- Number Lines

They use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line.



Number will be embedded throughout the curriculum and opportunities for explorative play within provision.

Children at the expected level of development will:

Number

- Have a deep understanding of number to 10, including the composition of each number;
- Subitise (recognise quantities without counting) up to 5;
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Numerical Patterns

- Verbally count beyond 20, recognising the pattern of the counting system;
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Subtraction

Year 1: Subtract from numbers up to 20.

Key Skills for subtraction in Year 1:

- Given a number, say one more or one less.
- Count to and over 100, forward and back, from any number.
- Represent and use subtraction facts to 20 and within 20.
- Subtract with one-digit and two-digit numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictures.
- Use subtraction to solve missing number problems.
- Read and write numbers from 0 – 20 in numerals and words.

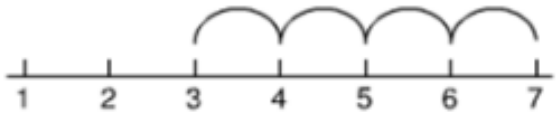
Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer, how many less than, most, least, count back, how many left, how much less is

Children consolidate understanding of subtraction practically, showing subtraction on different concrete materials and in familiar contexts. They are introduced to more formal recording methods and learn to read, write and interpret number sentences with – and = signs.

- Dienes
- Place Value Counters
- Bar Models
- Part-Whole Models
- Cubes
- Numicon
- Hundred Squares
- Rekenrek

Subtract by taking away:

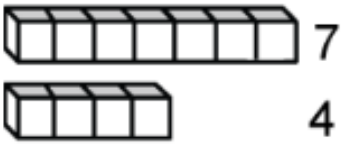


7 – 4 = 3

Count back in ones on a numbered number line to take away, with numbers up to 20

Model subtraction using hundred squares and numbered number lines/tracks and practically.

Finding the difference:



'Seven is 3 more than four.'

'I am 2 years older than my sister'

This will be introduced practically with the language **'find the distance between'** and '**how many more?**' in a range of familiar contexts.

Mental subtraction:

Children should start recalling subtraction facts up to **and within** 10 and 20.

Subtraction

Year 2: Subtract with 2-digit numbers.

Key Skills for subtraction in Year 2:

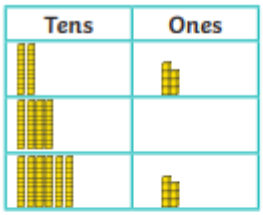
- Recognise the place value of each digit in a two-digit number and how to use place value to solve subtractions.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, number lines and hundred squares.
- Subtract mentally, including a two-digit number and units, a two-digit number and tens and two, two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order (is not commutative).
- Recognise and use the inverse relationship between addition and subtraction, using this to check calculations and solve missing number problems.
- Solve simple subtraction problems in a range of contexts using concrete objects, pictorial representations and also by applying increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.
- To use the symbols – and =.

Key Vocabulary

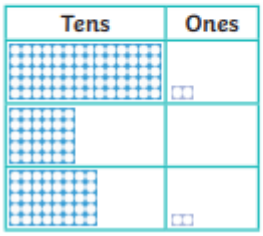
Equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer, how many less than, most, least, count back, how many left, how much less is, difference, count on, strategy, partition, tens, ones, column subtraction

Children may start their learning through the use of concrete and pictorial methods. They may use a range of equipment, such as:

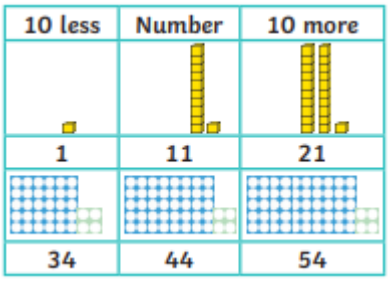
- Dienes
- Place Value Counters
- Bar Models
- Part-Whole Models
- Cubes
- Numicon
- Hundred Squares
- Rekenrek



27
+ 40
67

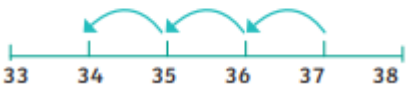


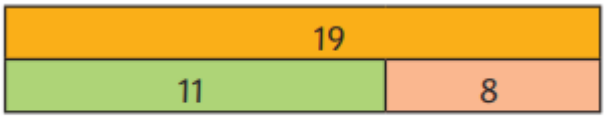
72
- 30
42



30	40	50	60	70	80
47	57	67	77	87	97

37 – 1 = 36
37 – 2 = 35
37 – 3 = 34





19 – 8 = 11 can be checked using 8 + 11 = 19

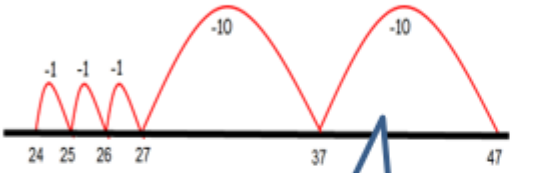
Subtract on a number line by counting back, aiming to develop mental subtraction skills.

This strategy will be used for:

- **2-digit numbers subtract 1-digit numbers** (by taking away/counting back) e.g. 38—7
- **2-digit numbers subtract tens** (by taking away/counting back) e.g. 48—30
- **Subtracting pairs of 2-digit numbers** (see below:)

Subtracting pairs of 2-digit numbers on a blank number line:

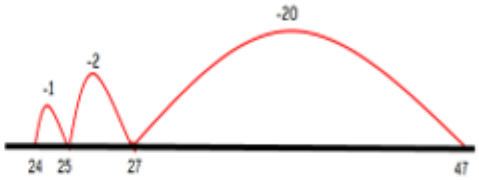
47 - 23 = 24 Partition the second number and subtract it in tens and units, as below:



Then subtract units.

Subtract tens first.

Move towards more efficient jumps back, as below:



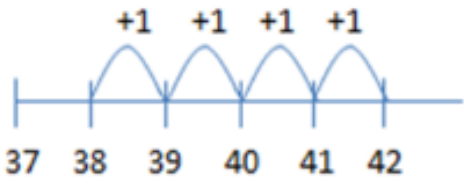
Combine methods with use of a hundred square to reinforce understanding of number value and order.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Teaching children to **bridge through ten** can help them to become more efficient.

Mental strategy - subtract numbers close together by **counting on:**

42 – 38 = 4



Start with the smaller number and count on to the largest.

Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

Next Step: Children who are confident and accurate with place value move to:

Compact method without exchanging or regrouping numbers.

Ensure that children understand the place value of the digits. When modelling, make sure children understand that it is 9 ones – 7 ones = 2 ones and then 3 tens – 2 tens = 1 ten (Not 3 – 2 = 1)

Subtraction

Year 3: Subtract with 2 and 3-digit numbers.

Key Skills for subtraction in Year 3:

- Recognise the place value of each digit in a 3-digit number.
- Find 10 or 100 more or less than a given number.
- Subtract mentally:
    - A 3-digit number and ones. (E.g. 345 – 8)
    - A 3-digit number and tens. (E.g. 567 – 90)
    - A 3-digit number and hundreds. (875 – 500)
- Subtract numbers with up to 3-digits using the formal method of columnar subtraction.
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems, using number facts, place value and more complex subtraction.

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer, how many less than, most, least, count back, how many left, how much less is __?, difference, count on, strategy, partition, hundreds, tens, ones, exchange/regroup, decrease, value, digit, tens barrier, hundreds barrier, estimate, inverse

Children will continue to learn through the use of concrete and pictorial methods. They may use a range of equipment or approaches such as:

- Dienes
- Place value counters
- Bar models
- Part-whole models
- Numicon
- Hundred square
- Number lines
- Cubes

Counting On as a Mental Strategy

Continue to reinforce counting on as a strategy for close-together numbers (E.g. 121 – 118), and also for numbers that are 'nearly' multiples of 10 or 100 or £s, which make it easier to count on (E.g. 102 – 89, 131 – 79, or calculating change from £1)

Use Bar Models to Represent and Solve Problems
Such as:

Bar models can also be used to show that a part must be taken away from the whole.

Children who are not secure in their understanding of place value may need to use partitioned subtraction as a written method to work out a calculation.

STEP 1: introduce this method with examples where **no exchanging** is required.

89 - 35 = 54

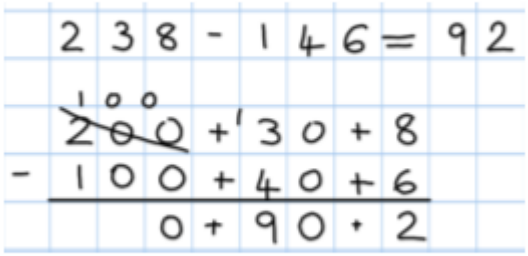
80 + 9
- 30 + 5
50 + 4

STEP 2: introduce 'exchanging" through subtraction.

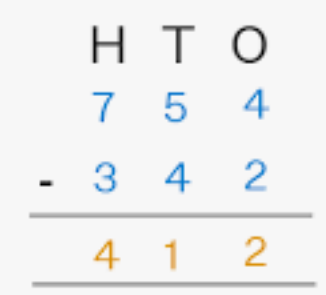
72 - 47 = 25

70 + 2 or 60 +12
- 40 + 7
20 + 5 = **25**

STEP 3: Once pupils are secure with the understanding of "exchanging", they can use the partitioned column method to subtract any 2 and 3-digit numbers.

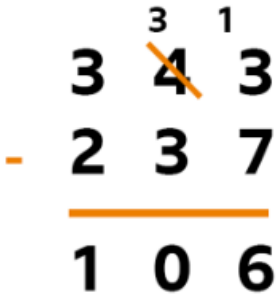


Once children are secure with their understanding of place value, they can move onto formal written methods of columnar subtraction. Starting without exchanging/regrouping and then moving onto this.



When modelling how to use this method, ensure that children are aware that it is 4 ones – 2 ones = 2 ones, 5 tens – 4 tens = 1 ten (not 5 – 4 = 1) and 7 hundreds – 3 hundreds = 4 hundreds (not 7 – 3 = 4).

Remember to keep to the same language as when modelling without exchanging or regrouping.



Subtraction

Year 4: Subtract numbers with up to 4-digits.

Key Skills for subtraction in Year 4:

- Recognise the place value of each digit in a 4-digit number.
- Find 1000 more or less than a given number.
- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 or 1000.
- Subtract numbers with up to 4-digits using the formal method of columnar subtraction.
- Estimate and use inverse operations to check answers.
- Solve subtraction 2-step word problems, choosing which operations and methods to use and why.
- Solve number and practical problems that involve all of the above, with increasingly large positive numbers. (Do we need this in here as it is a place value objective?)

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer, how many less than, most, least, count back, how many left, how much less is __?, difference, count on, strategy, partition, thousands, hundreds, tens, ones, exchange/regroup, decrease, value, digit, tens barrier, hundreds barrier, thousands barrier, inverse, estimate

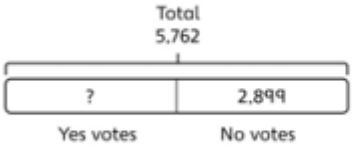
Children will continue to learn through the use of concrete and pictorial methods. They may use a range of equipment or approaches such as:

- Dienes
- Place value counters
- Bar models
- Part-whole models
- Numicon
- Hundred square
- Number lines
- Cubes

Use place value grids to support mental methods where appropriate.

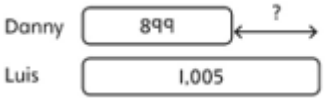
7,646 – 40 = 7,606

Use bar models to represent subtractions where a part needs to be calculated.

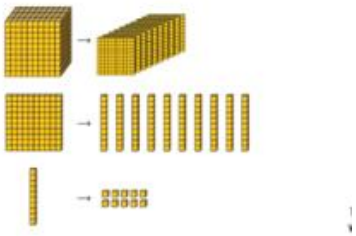


I can work out the total number of Yes votes using 5,762 – 2,899.

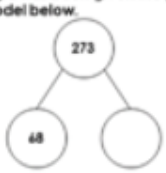
Bar models can also represent 'find the difference' as a subtraction problem.



Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.



1a. Georgia has been given the part-whole model below.

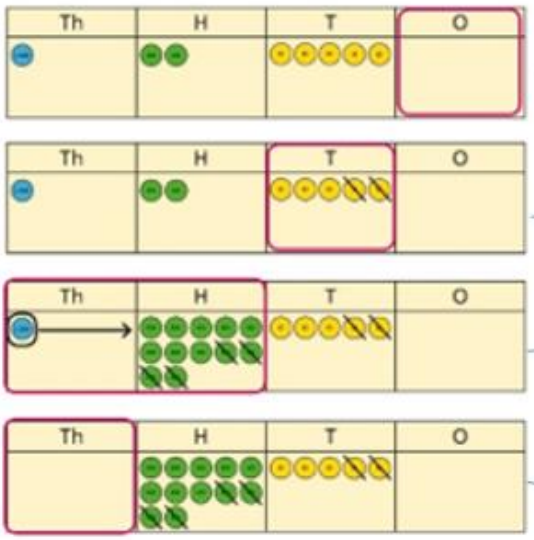


She thinks the type of calculation is subtraction and the answer is 215.

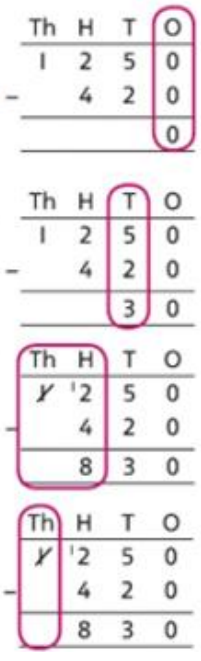
Do you agree? Explain your answer.

1250 – 420 = **830**

Represent place value equipment on a place value grid to subtract, including exchanges where needed.



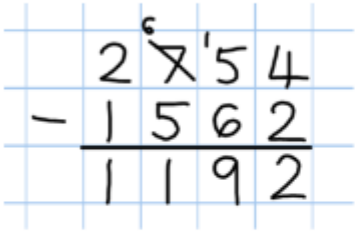
Use column subtraction, with understanding of the place value of any exchange required.



Mental strategies:

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Compact column subtraction



To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it.

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved — mental, counting on, counting back or written method.

Money

	£	$^{4}\not{5}$	$^{12}\not{3}$	.	$^{1}2$	2
-	£	2	5	.	6	1
	£	2	7	.	6	1

Subtraction

Year 5: Subtract numbers with at least 5-digit numbers. Including money, measures and decimals.

Key Skills for subtraction in Year 5:

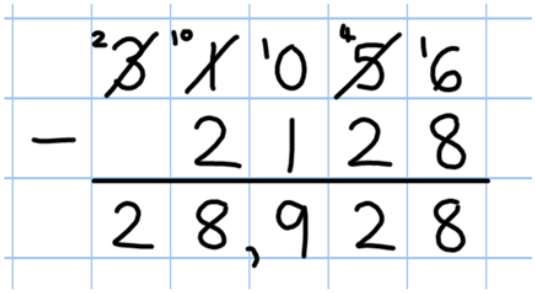
- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, difference/how many between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, millions, hundred thousands, ten thousands, thousands, hundreds, tens, ones, exchange, decrease, hundreds, value, digit, ten thousands barrier, hundred thousands barrier, inverse, tenths, hundredths, decimal point, decimal.

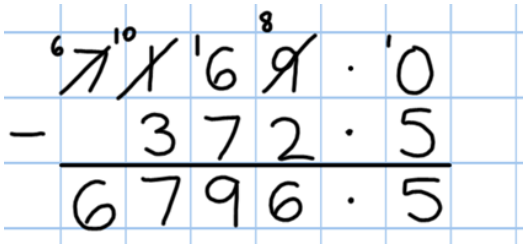
In Year 5 pupils will carry on building on the methods they have been learning in previous years.

Subtracting with larger integers.



Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

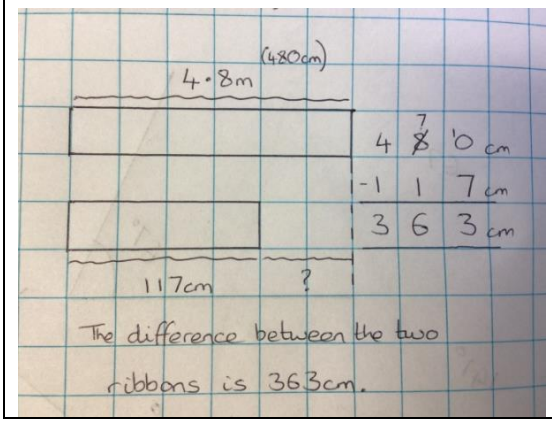
Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.



Create lots of opportunities for subtracting and finding differences with money and measures.

Add a 'zero' in any empty decimal places to aid understanding of what to subtract in that column (place holder).

- Pupils can use pictorial methods to help them solve subtraction word problems.



Sam has one piece of ribbon which is 4.8m, Lucy has a piece of ribbon which is 117cm. What is the difference between the two lengths?

Children should:

Understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

Subtraction

Year 6: Subtract with increasing large and more complex numbers and decimal values.

Key Skills for subtraction in Year 6:

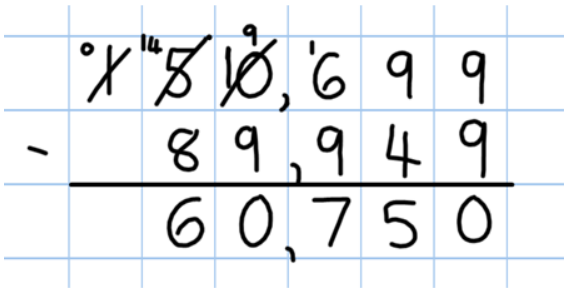
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

Key Vocabulary

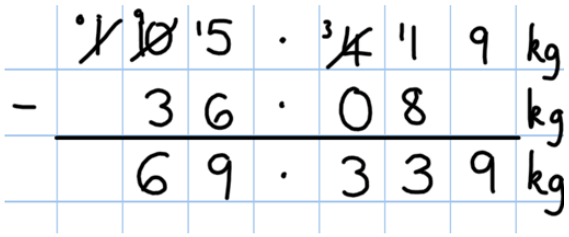
Equal to, take, take away, less, minus, subtract, leaves, difference between, how many more, how many fewer, how many less than, most, least, count back, how many left, how much less is __?, difference, count on, strategy, partition, millions, hundred thousands, ten thousands, thousands, hundreds, tens, ones, exchange/regroup, decrease, value, digit, tens barrier, hundreds barrier, thousands barrier, ten thousands barrier, hundred thousands barrier, inverse, estimate

In Year 6 pupils will carry on building on the methods they have been learning in previous years.

- Using the compact column method to subtract more complex integers.

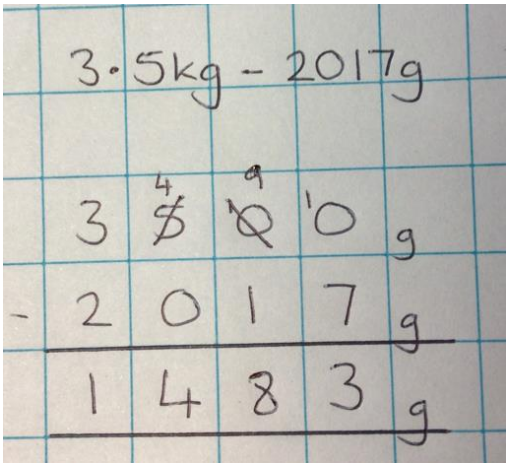


- Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.



Empty decimal places can be filled with **zero** to show the place value in each column (place holder).

- Subtracting mixed units of measures.



Children should:
be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

Multiplication

EYFS: Children to have a deep understanding of number to 10, the relationships between them and the patterns within those numbers.

Key Skills for Multiplication in EYFS:

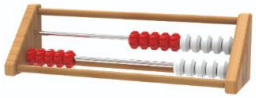
- Explore and represent patterns within numbers up to 10.
- Children to verbally count beyond 20, recognising the pattern of the counting system.
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Key Vocabulary

Double, groups of, lots of, repeated addition.

They use concrete and pictorial representation to record their calculations.

- Children use a wide range of concrete objects to count and double.
    - Everyday objects such as: small vehicles, small animals, jewels and teddy bears.
    - Bead strings
    - Cubes
    - Numicon
    - Rekenrek



They will count on in twos using a bead string and number line.

They understand doubling as repeated addition. 2 + 2 = 4

They should also be encouraged to use a range of representations to help them visualise Maths in a pictorial way. This will allow them to see numbers in different contexts.

- Rekenreks
- Bead strings
- Number lines
- Cubes

Some children may be able to represent their calculations using symbols and numbers within a written calculation.

Number will be embedded throughout the curriculum and opportunities for explorative play within provision.

Children at the expected level of development will:

Number

- Have a deep understanding of number to 10, including the composition of each number;
- Subitise (recognise quantities without counting) up to 5;
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Numerical Patterns

- Verbally count beyond 20, recognising the pattern of the counting system;
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Year 1: Multiply with concrete objects, repeated addition, arrays and pictorial representations

Key Skills for multiplication in Year 1:

- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations, repeated addition and arrays with the support of the teacher.
- Begin to understand doubling using concrete objects and pictorial representations.

Key Vocabulary

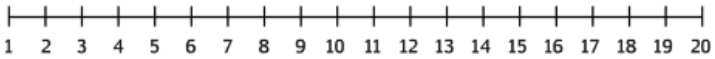
Multiplication, multiply, groups of, lots of, array, count

They should start their learning journey using manipulatives such as:

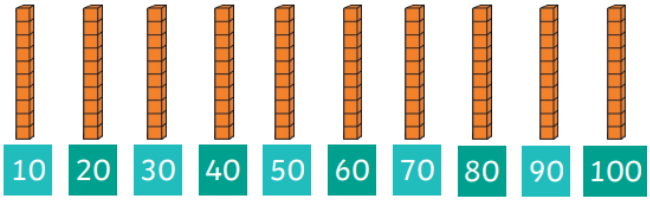
- A wide range of counting equipment.
  - Everyday objects such as: small vehicles, small animals, jewels and teddy bears.
  - Bead strings
  - Cubes

They should also be encouraged to use a range of representations to help them visualise Maths in a pictorial way. This will allow them to see numbers in different contexts.

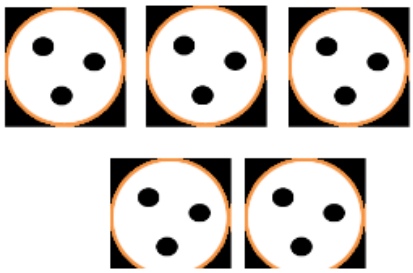
- Counters
- Cubes
- Hundred Squares
- Number lines
- Rekenrek
- Multiplication Squares



- Give children experience of counting equal groups of objects in 2s, 5s and 10s.

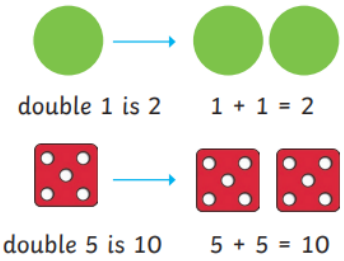


How many legs will 3 teddies have? 2 + 2 + 2 = 6



How many dots? 3 + 3 + 3 + 3 + 3 = 15

- Present practical problem solving activities involving counting equal sets or groups.
- Make doubles.



Multiplication

Year 2: Multiply with concrete objects, repeated addition, arrays and pictorial representations

Key Skills for multiplication in Year 2:

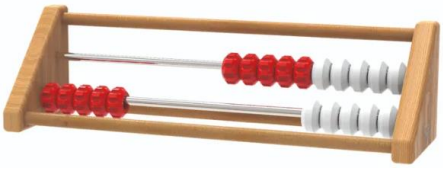
- Count in steps of 2, 3 and 5 from zero and in 10s from any number.
- Recall and use multiplication facts for 2, 5 and 10 multiplication tables, including recognising odds and evens.
- Calculate mathematical statements for multiplication and division and write them using the multiplication (x), division (÷) and equals (=) signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve a range of problems involving multiplication and division, using arrays, repeated addition and multiplication and division facts, including problems in context.
- Use multiplication and division facts for 2, 5 and 10 to make deductions outside know multiplication facts.
- Solve word problems involving multiplication and division with more than one step.
- Recognise the relationship between addition and subtraction and rewrite addition statements as simplified multiplication statements.

Key Vocabulary

Multiplication, multiply, multiplied by, multiplication tables, times tables, groups of, lots of, array, count, times, repeated addition, column, row, commutative, sets of, equal groups, times, as big as, once, twice, three times, mentally, orally

In Year 2, children should continue their journey using manipulatives to support pictorial representations alongside written methods. This will develop their understanding of multiplication methods. They should have access to equipment such as but not limited to, the following:

- Everyday objects such as: small vehicles, small animals, jewels
- Bead strings
- Cubes
- Counters
- Hundred squares to help identify patterns
- Number lines
- Multiplication squares – including partially filled squares that the children can complete
- Rekenrek
- Numicon



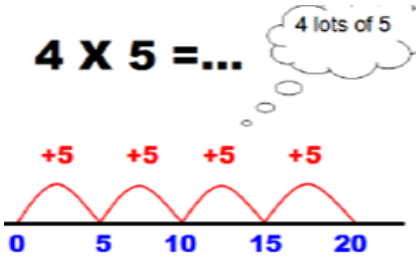
- Recognise equal groups and use this to work out multiplications. 5 x 3 =

5 equal groups with 3 in each group

- Add equal groups.

2 + 2 + 2 + 2 = 8 apples

- Use repeated addition on a number line. E.g. starting from zero, make equal jumps along a number line to work out multiplication facts and write multiplication statements.



- Use arrays to help teach children to understand the commutative law of multiplication and give example.

4 rows of 10 = 40
10 columns of 4 = 40

- Children should begin to recall multiplication facts for 2, 5 and 10 times tables through practice in counting and understanding of the operation.

Multiplication

Year 3: Multiply 2 digits by a single digit number

Key Skills for multiplication in Year 3:

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g. using commutativity (**4 × 12 × 5** = 4 × 5 × 12 **= 20 × 12** = 240) and for missing number problems x 5 = 20, 3 x = 18, x = 32

Key Vocabulary

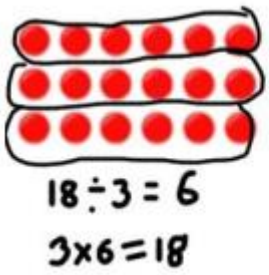
groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times...,partition, grid method, multiple, product, tens, units, value

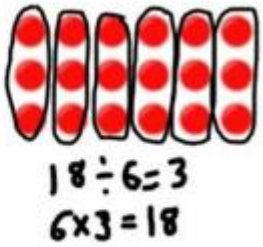
Children will continue to learn through the use of concrete and pictorial methods. They may use a range of equipment or approaches such as:

- Dienes
- Place Value counters
- Numicon
- Multiplication Squares
- Number lines
- Cubes

Children understand the link between multiplication and division and use physical objects to find related facts.

Children represent an array pictorially and then multiplication and division facts by sorting into equal groups.

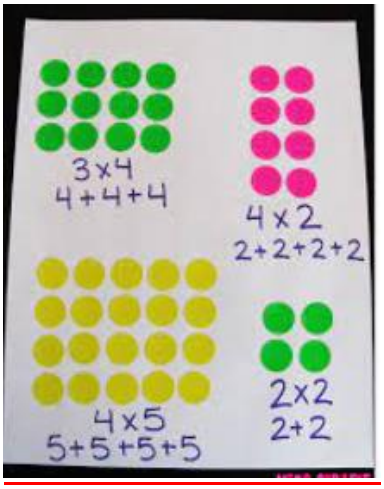




Children apply their understanding of inverse relationships to write related multiplication and division statements.

3 x 6 = 18	18 = 3 x 6
6 x 3 = 18	18 = 6 x 3
18 ÷ 3 = 6	6= 18 ÷ 3
18 ÷ 6 = 3	3= 18 ÷ 6

Step 1 – Revise equal groups, arrays and repeated addition from Year 2.



Step 2

24 x 3 = 72

Use of unit cubes

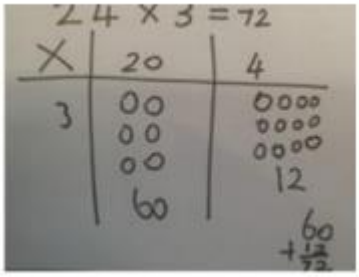
use of base 10

Use of place value counters

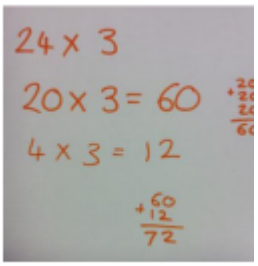
finding the total

Step 3

Children show their understanding by representing the calculation in a grid using their own pictorial representation.



Children to use jottings to partition and multiply each part.

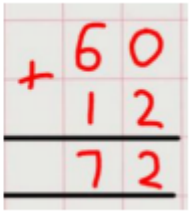


Step 4

Children use the grid method to partition and multiply each number.

24 x 3 = 72

X	20	4
3	60	12



Children should apply this knowledge to problem solving such as:

There are 3 balloons in a packet. There are 24 packets in a box. How many balloons are there altogether in a box?

Multiplication

Year 4: Multiply 2 and 3 digits by a single digit number

Key Skills for multiplication in Year 4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12.**
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39x7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000

Key Vocabulary

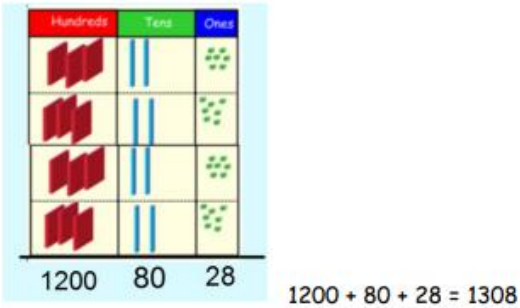
groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, inverse

Children will continue to learn through the use of concrete and pictorial methods. They may use a range of equipment or approaches such as:

- Dienes
- Place Value counters
- Numicon
- Multiplication Squares
- Number lines
- Cubes

Step 1

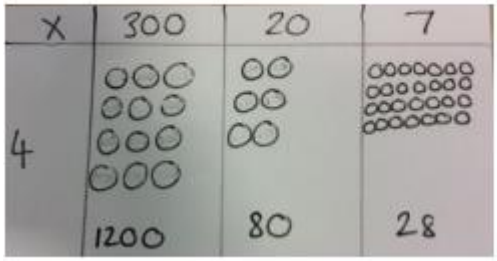
Children should recap on the grid method from Year 3 and represent calculations using manipulatives such as place value counters or base ten equipment.



Step 2

Children show their understanding by representing the calculation in a grid using their own pictorial representation and by using jottings to partition and multiply.

327 x 4 = 1308

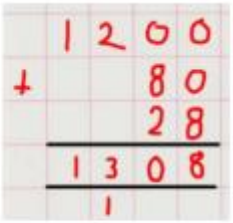


Step 3

Children continue to use the grid method to multiply each part.

327 x 4 = 1308

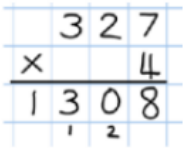
X	300	20	7
4	1200	80	28



1200 + 80 + 28 = 1308

Step 4

Children move to using the condensed method of short multiplication. They carry below the line.



Multiplication

Year 5: Multiply up to 4-digits by 1 or 2 digits.

Key Skills for multiplication in Year 5:

- Identify multiples and factors, using knowledge of multiplication tables to 12x12.
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube numbers and their notation.
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Key Vocabulary

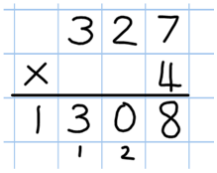
groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short/long multiplication, 'carry'.

Introducing column multiplication:

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method.
- Children need to be taught to approximate first, e.g. for **72 x 38, they will use rounding: 72 x 38** is approximately 70 x 40 = **2800**, and use the approximation to check the reasonableness of their answer.

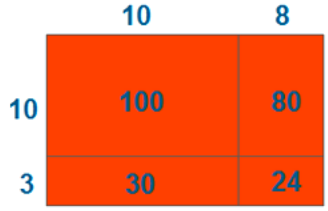
Short multiplication for multiplying by a single digit:

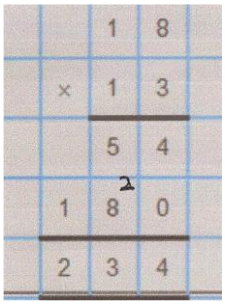
x	300	20	7
4	1200	80	28



Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits:





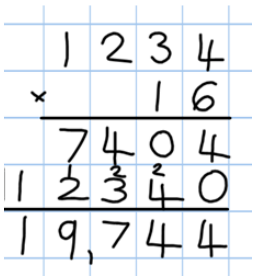
18 x 3 on the 1st row

(8 x 3 = 24, carrying the 2 for twenty, then 10 x 3).

18 x **10** on the 2nd row. Put a zero in units first, then say 8 x 10, and 10 x 10.

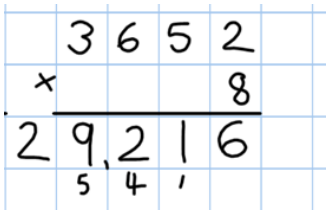
The grid could be used to introduce long multiplication, as the relationship can be seen in the answers in each row.

Moving towards more complex numbers:



(1234x6)

(1234x10)



Multiplication

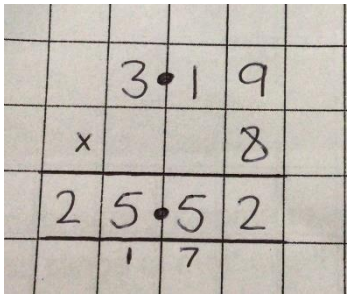
Year 6: Short and long multiplication as in Y5, and multiply decimals with up to 2 decimal places by a single digit.

Key Skills for multiplication in Year 6:

- Recall multiplication facts for all times tables up to 12 x 12 (as Y4 and Y5).
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.

Key Vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, __times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, 'carry', tenths, hundredths, decimal.



Line up the decimal points in the question and the answer.

This works well for multiplying money (£.p) and other measures.

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**.
- Multiply money and measures and **multiply decimals with up to 2 decimal places by a single digit.**
- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number.**

Division

EYFS: Children to have a deep understanding of number to 10, the relationships between them and the patterns within those numbers.

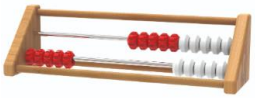
Key Skills for Division in EYFS:

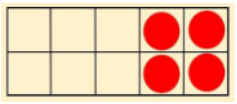
- Explore and represent patterns within numbers up to 10.
- Children to verbally count beyond 20, recognising the pattern of the counting system.
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Key Vocabulary

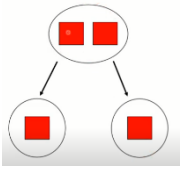
Divide, share, even groups, equal, same.

- Children use a wide range of concrete objects to count and share equally into 2 groups.
    - Everyday objects such as: small vehicles, small animals, jewels and teddy bears.
    - Bead strings
    - Cubes
    - Numicon
    - Rekenrek
    - Counters on Ten frames





They should also be encouraged to use a range of representations to help them visualise Maths in a pictorial way. This will allow them to see numbers in different contexts.



- Rekenreks
- Place Value Counters
- Cubes
- Part-Whole Models

6 cakes shared between 2 people each person gets 3 cakes. 6 ÷ 2 = 3 They count a set of objects and halve them by making two equal groups.

They understand sharing and halving as dividing by 2. They will begin to use objects to make groups of 2 from a given amount.

They use concrete and pictorial representation to record their calculations.

Some children may be able to represent their calculations using symbols and numbers within a written calculation.

Number will be embedded throughout the curriculum and opportunities for explorative play within provision.

Children at the expected level of development will:

Number

- Have a deep understanding of number to 10, including the composition of each number;
- Subitise (recognise quantities without counting) up to 5;
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

Numerical Patterns

- Verbally count beyond 20, recognising the pattern of the counting system;
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Division

Year 1: Group and share small quantities.

Key Skills for multiplication in Year 1:

- Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Key Vocabulary

Division, divide, equal groups, share, share equally

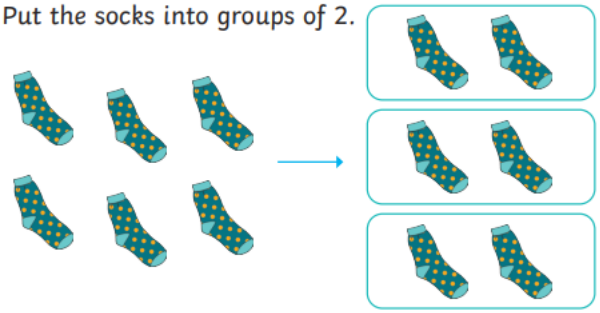
In Year 1, children should start their journey using manipulatives to support pictorial representations alongside written methods. This will develop their understanding of division. They should have access to equipment such as but not limited to, the following:

- Everyday objects such as: small vehicles, small animals, jewels
- Bead strings
- Cubes
- Counters
- Number lines

Using objects, diagrams and pictorial representations to solve problems involving both grouping and sharing.

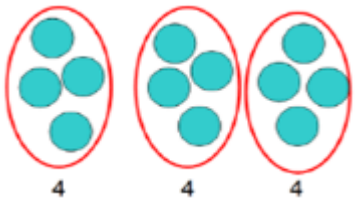
Grouping/ Group Equally

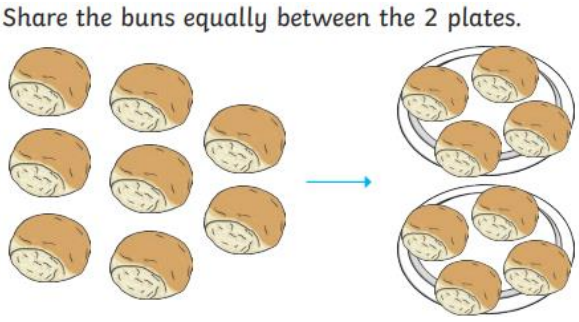
How many groups of 4 can be made with 12 stars?



Sharing/ Share Equally

What is 12 shared between 3?





Division

Year 2: Group and share using the ÷ and = sign.

Key Skills for multiplication in Year 2:

- Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Key Vocabulary

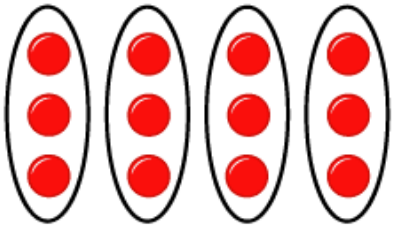
Division, divide, equal groups, share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

In Year 2, children should continue their journey using manipulatives to support pictorial representations alongside written methods. This will develop their understanding of division. They should have access to equipment such as but not limited to, the following:

- Everyday objects such as: small vehicles, small animals, jewels
- Bead strings
- Cubes
- Counters
- Number lines

Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays



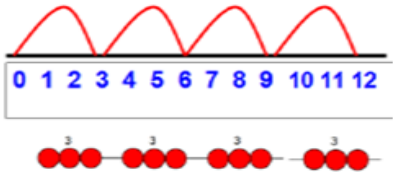
12 ÷ 3 = 4

This represents **12 ÷ 3, posed as** how many groups of 3 are in 12?

Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

Grouping Using a Number Line

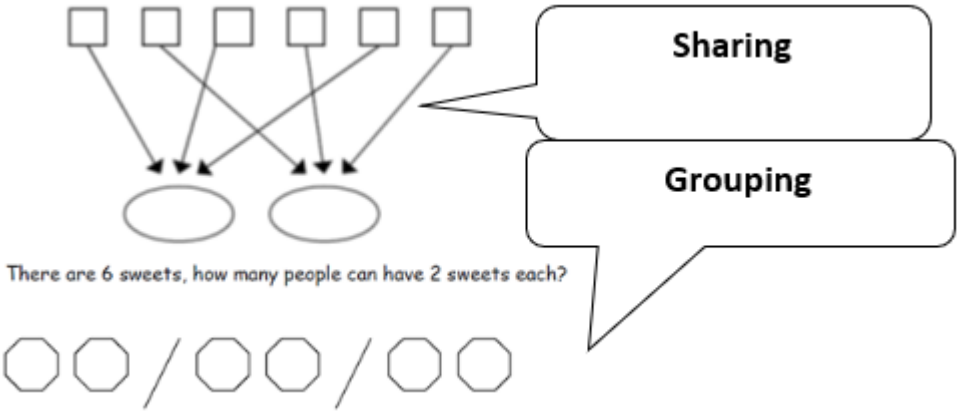
Group from zero in equal jumps of the divisor to find out "how many groups of _ in _ ?". Pupils could and using a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12?" **This is an important method to develop understanding of division as grouping.**



12 ÷ 3 = 4

Know and Understand Sharing and Grouping

6 sweets shared between 2 people. How many do they get each?



Children should be taught to recognise whether a problem requires sharing or grouping.

Division

Year 3: Divide 2 digits by a single digit number.

Key Skills for division in Year 3:

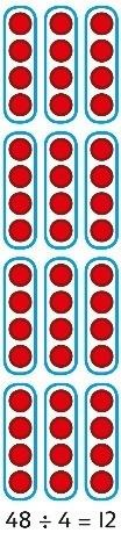
- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Key Vocabulary

share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', multiple

Step 1 – Calculate divisions using known tables

24 divided into groups of 8.
There are 3 groups of 8.



48 ÷ 4 = 12

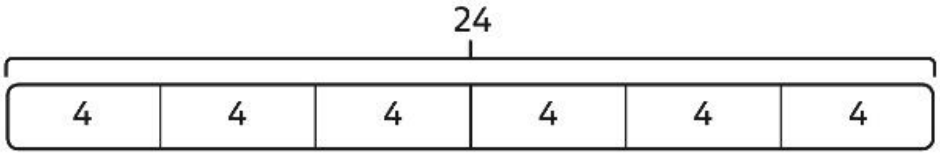
48 divided into groups of 4.
There are 12 groups.

4 × 12 = 48
48 ÷ 4 = 12

I need to work out 30 shared between 5.

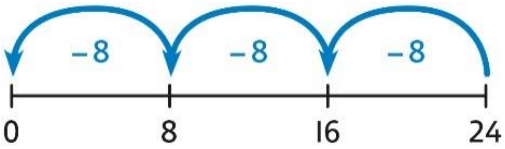
I know that 6 × 5 = 30
so I know that 30 ÷ 5 = 6.

A bar model may represent the relationship between sharing and grouping.



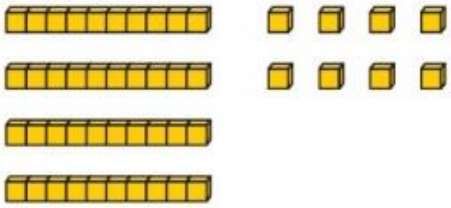
24 ÷ 4 = 6
24 ÷ 6 = 4

Step 2 - Children understand how division is related to both repeated subtraction.

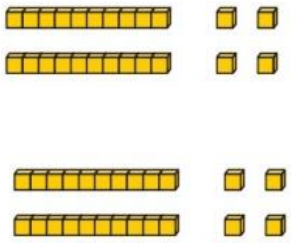


24 ÷ 8 = 3

Step 3 - Children explore dividing 2-digit numbers by using place value equipment. (no remainders)

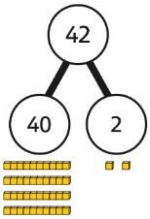


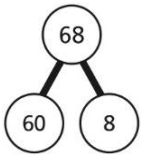
48 ÷ 2 = ?



48 ÷ 2 =24

Step 4 - Children explore which partitions support particular divisions. (no remainders)



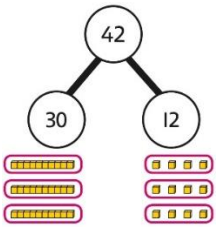


60 ÷ 2 = 30
8 ÷ 2 = 4
30 + 4 = 34
68 ÷ 2 = 34

Step *5* – Children explore how to divide differently.

42 ÷ 3 =

I need to partition 42 differently to divide by 3.



42 = 30 + 12

42 = 30 + 12

30 ÷ 3 = 10

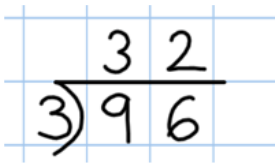
12 ÷ 3 = 4

10 + 4 = 14

42 ÷ 3 = 14

Step 6

Once children are secure with division as grouping and demonstrate this using number lines, arrays, etc. Short division for larger 2-digit numbers should be introduced.



Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:

- How many 3‟s in 9? = 3, and record it above the **9 tens**.
- How many 3‟s in 6? = 2, and record it above the **6 units**

Division

Year 4: Divide up to 3 digits by a single digit number (with remainders only within the calculation)

Key Skills for division in Year 4:

- Recall multiplication and division facts for all numbers up to 12 x 12.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Key Vocabulary

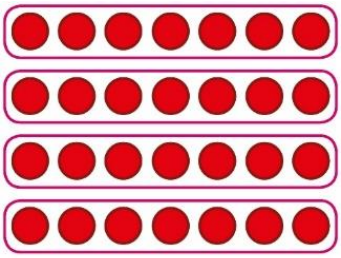
share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', multiple, remainder, divisible by, factor

Step 1 - Continue to develop mental skills and understanding:

Use objects to explore families of multiplication and division facts.

4 × 6 = 24
24 is 6 groups of 4
24 is 4 groups of 6
24 divided by 6 is 4
24 divided by 4 is 6

Represent divisions using an array.



28 ÷ 7 = 4

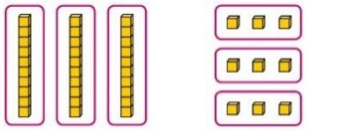
Understand families of related multiplication and division facts.

I know that 5 × 7 = 35 so I know all these facts:

5 × 7 = 35
7 × 5 = 35
35 = 5 × 7
35 = 7 × 5
35 ÷ 5 = 7
35 ÷ 7 = 5
7 = 35 ÷ 5
5 = 35 ÷ 7

Step 2 - Use Base 10 to partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate. (no remainders)

39 ÷ 3 = ?

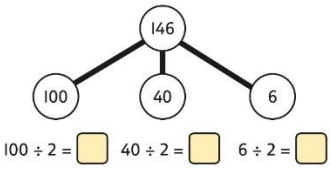


39 = 30 + 9

30 ÷ 3 = 10
9 ÷ 3 = 3
39 ÷ 3 = 13

Step 3 - Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate. (no remainders)

142 ÷ 2 = ?

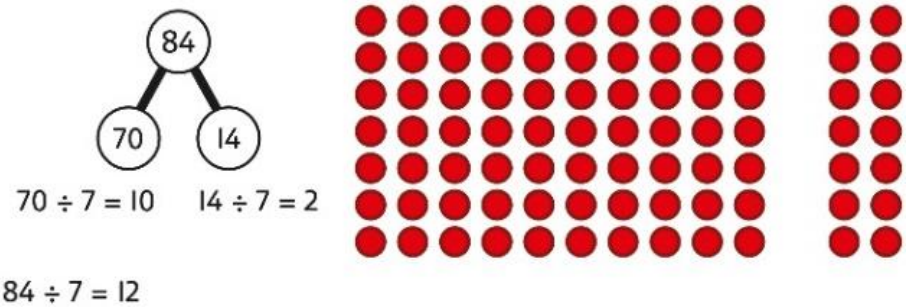


100 ÷ 2 = 50
40 ÷ 2 = 20
6 ÷ 2 = 3
50 + 20 + 3 = 73
142 ÷ 2 = 73

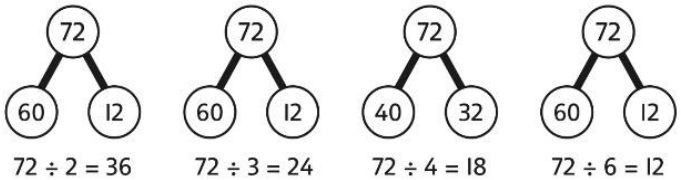
Represent how to partition flexibly where needed. (no remainders)

84 ÷ 7 = ?

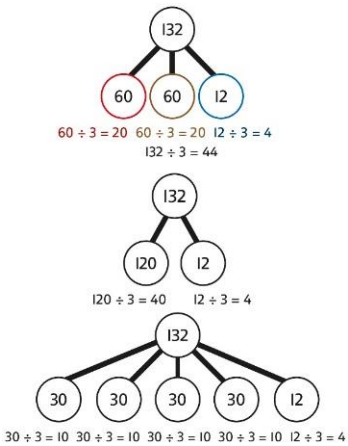
I will partition into 70 and 14 because I am dividing by 7.



Make decisions about appropriate partitioning based on the division required.



Understand that different partitions can be used to complete the same division.

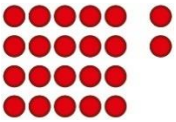


Step 4 – Understand remainders using Base 10, place value counters and part whole models

Use equipment to understand that a **remainder** occurs when a set of objects cannot be divided equally any further.

There are 13 sticks in total. *There are 3 groups of 4, with 1 remainder.*

Use images to explain remainders.



22 ÷ 5 = 4 remainder 2

Use place value equipment to understand the concept of remainder.

29 ÷ 2 =

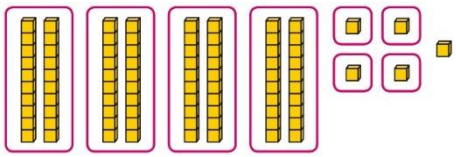
Make 29 from place value equipment.
Share it into 2 equal groups.

There are two groups of 14 and
1 remainder. *29 ÷ 2 = 14 remainder 1*

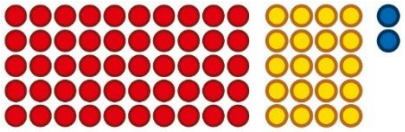
85 ÷ *4 =*

85 shared into 4 equal groups

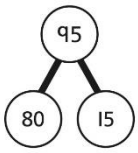
There are 24, and 1 that cannot be shared.



Represent the remainder as the part that cannot be shared equally.



72 ÷ 5 = 14 remainder 2
Understand how partitioning can reveal remainders of divisions.



80 ÷ 4 = 20
12 ÷ 4 = 3

95 ÷ 4 = 23 remainder 3

Step 5 - Partition to divide, understanding the remainder in context.

67 children try to make 5 equal lines.

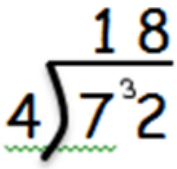
67 = 50 + 17
50 ÷ 5 = 10

17 ÷ 5 = 3 remainder 2
67 ÷ 5 = 10 + 3 = 13 remainder 2

There are 13 children in each line and
2 children left out.

Step 6 - Use formal calculations to calculate:

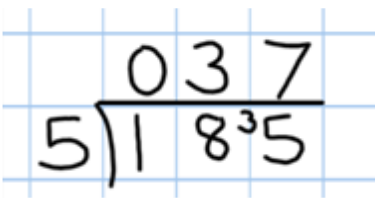
Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation and be taught to 'carry' the remainder onto the next digit.

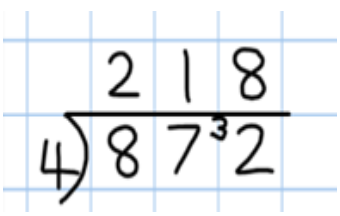


Limit numbers to **NO** remainders in the final answer, but with remainders occurring within the calculation.

Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder)** but must understand how to calculate remainders, using this to 'carry' remainders within the calculation process (see example).

Step 7 - Pupils move onto dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage**.**





When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always 'carry' the number (1) over to the next digit as a remainder.

Include money and measure contexts when confident.

Division

Year 5: **Divide up to 4 digits by a single digit,** including those with remainders.

Key Skills for division in Year 5:

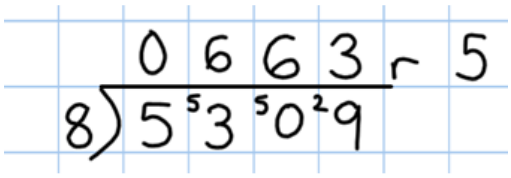
- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving division where larger numbers are decomposed into their factors.
- Divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = 24 ½ = 24.5 ≈ 25).
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Key Vocabulary

Share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime).

Year 5: Divide up to 4 digits by a single digit, including those with remainders.

Short division, including remainder answers:



Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it** i.e. as a fraction, a decimal, or as a rounded number or value, depending upon the context of the problem.

The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved.

See Y6 for how to continue the short division to give a decimal answer for children who are confident.

Include money and measure contexts.

Division

Year 6: **Divide at least 4 digits by both single-digit and 2-digit numbers** (including decimal numbers and quantities).

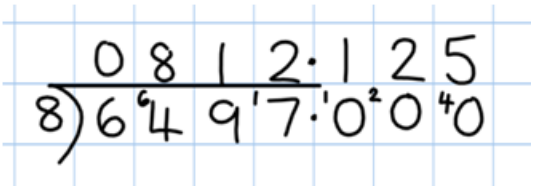
Key Skills for division in Year 6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

Key Vocabulary

Share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime), common factor.

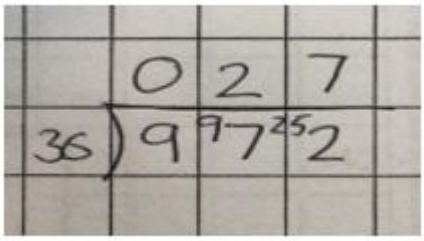
Short division, for dividing by a single digit: e.g. 6497 ÷ 8



Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as **r1**, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Introduce long division by considering multiples and chunking for dividing by 2 digits.



Pupils are taught to consider their 36 times tables to help with a short division method using their knowledge of multiples.

30 + 6 = 36

60 + 12 = 72

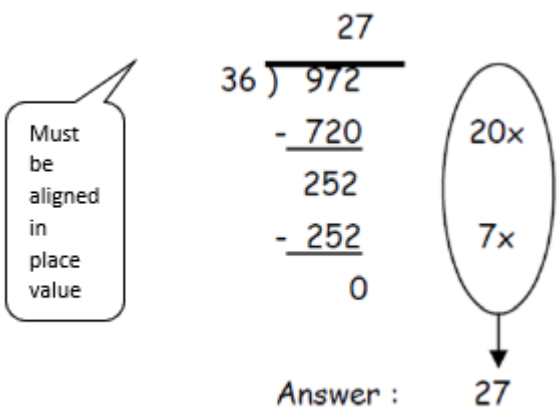
90 + 18 = 108

120 + 24 = 144

150 + 30 = 180

180 + 36 = 216

210 + 42 = 252



Find out "How many 36s are in 972?" by subtracting 'chunks' of 36, until zero is reached (or until there is a remainder).

Teach pupils to write a '**useful list**' first at the side that will help them decide what chunks to use, e.g.:

'Useful' list: 1x = 36
10x =360
100x = 3600

Where **remainders** occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

Introduce the method in a simple way by limiting the choice of chunks to "Can we use 10 lots? Can we use 100 lots?" As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their 'useful' lists.